MATA30 Fall 2018
Tutorial: 0007
Quiz 1

Name: ____________________

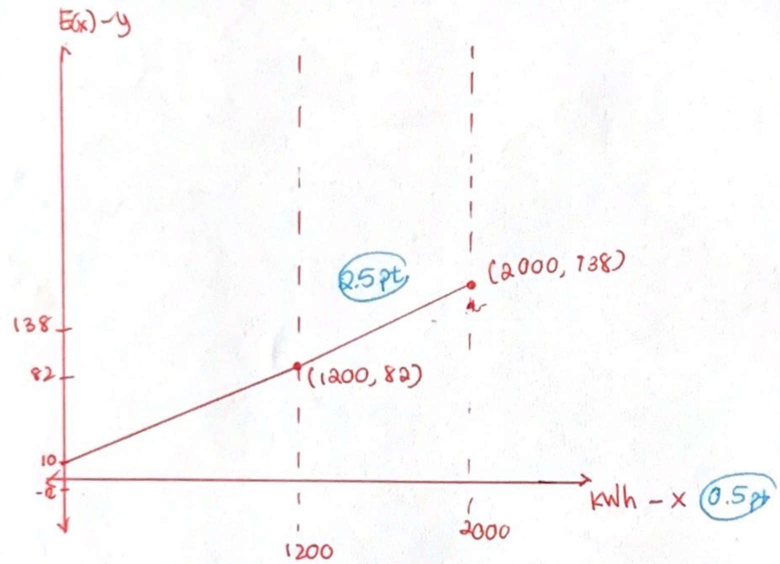
Student Number: ____________________

1. (10 Marks) An electricity company charges its customers a base rate of \$10 a month, plus 6 cents per kilowatt-hour (kWh) for the first 1200 kWh and and 7 cents per kWh for all usage over 1200kWh. Express the monthly cost E as a function of the amount x of electricity used. Then graph the function E for $0 \le x \le 2000$.

$$E(x) = \begin{cases} 10 + 0.06x & \text{1pt} \\ 10 + (0.06)(1200) + 0.07(x-1200) & \text{2pt} \end{cases} \quad \begin{cases} 10 + 0.06x \ \ \text{2pt}, & 0 \le x \le 1200 \\ 82 + 0.07(x-1200), & x > 1200 \ \ \text{2pt} \end{cases}$$

↳ OR $-2 + 0.07x$

→ 5 eq
→ 2 bounds
→ 3 graph.



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QUIZ 2

NAME: ____________________

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1. (5 MARKS) If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$ and $f(f^{-1}(2))$

* $f(1) = 3 \rightarrow \underline{f^{-1}(3)} = 1$ 3

* $\underline{f(f^{-1}(2))} = 2$ 2

2. (5 MARKS) Find the inverse of:

$$f(x) = \frac{4x-1}{2x+3}$$

$$y = \frac{4x-1}{2x+3}$$

$$\left.\begin{aligned}(2x+3)y &= 4x-1\\ 2xy + 3y &= 4x-1\\ 3y+1 &= 4x-2xy\\ 3y+1 &= x(4-2y)\\ x &= \frac{3y+1}{4-2y}\end{aligned}\right\}$$ 3 for process

$$\therefore f^{-1}(x) = \frac{3x+1}{4-2x}$$ 2

$$\underline{\text{or}} = \frac{-1-3x}{2x-4}$$

Quiz 3 Ans.

$$\lim_{x \to 3} (2x + |x-3|)$$

$x-3<0$

$$|x-3| \begin{cases} -(x-3), & \underline{x<3} \\ x-3, & \underline{x \geq 3} \end{cases}$$

$x-3 \geq 0$ (1)

(3)

$$\lim_{x \to 3^-} (2x-(x-3)) = \lim_{x \to 3^-} x+3 = 3+3 = 6$$

(1)

$$\lim_{x \to 3^+} (2x+x-3) = \lim_{x \to 3^+} 3x-3 = 9-3 = 6$$

(3)

(1)

$=$ $\therefore \lim_{x \to 3} (2x+|x-3|) = 6.$

(1)

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Quiz 4

Name: ______________________

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1. (10 Marks) Find the horizontal and vertical asymptotes of the following curve:

$$y = \frac{2x^2+1}{3x^2+2x-1}$$

$\begin{matrix} 3 & & -1 \\ 1 & & 1 \end{matrix}$

VA $y = \dfrac{2x^2+1}{(3x-1)(x+1)}$ 1

$x = \frac{1}{3}, -1$ possible Assymptotes.

$\lim_{x \to -1^-} f(x) = \infty$

$\lim_{x \to -1^+} f(x) = -\infty$

$\lim_{x \to 1/3^-} f(x) = -\infty$

4

$\lim_{x \to 1/3^+} f(x) = \infty$

$\therefore \; x = \frac{1}{3}, -1$ 2

HA $y = \dfrac{2x^2+1}{3x^2+2x-1}$

$\therefore y = \frac{2}{3}$ 2

$\lim_{x \to \pm\infty} y = \frac{2}{3}$ 1

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Quiz 5

Name: ______________________

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1. (3 Marks) Differentiate:

$$f(x) = \sqrt[3]{x}(2+x)$$

$$f'(x) = (x^{1/3})'(2+x) + (x^{1/3})(2+x)' \quad 2$$

$$= \frac{1}{3}x^{-2/3}(2+x) + x^{1/3} \quad 1$$

$$= \frac{2+x}{3\sqrt[3]{x^2}} + \sqrt[3]{x}$$

2. (7 Marks) Differentiate:

$$f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$$

$$f'(x) = \frac{1}{2}\left[x+\sqrt{x+\sqrt{x}}\right]^{-1/2}\left[x+\sqrt{x+\sqrt{x}}\right]' \quad 2$$

$$= \frac{1}{2}\left[x+\sqrt{x+\sqrt{x}}\right]^{-1/2}\left[1+\left[\frac{1}{2}(x+\sqrt{x})^{-1/2}\right]\left[x+\sqrt{x}\right]'\right] \quad 2$$

$$= \frac{1}{2}\left[x+\sqrt{x+\sqrt{x}}\right]^{-1/2}\left[1+\frac{1}{2}(x+\sqrt{x})^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)\right] \quad 3$$

$$= \cancel{\frac{1}{2}\left[x+\sqrt{x+\sqrt{x}}\right]^{-1/2}\left[\right.}$$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)\right]$$

$$\underline{\text{OR}} \quad \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[1+\frac{1}{2\sqrt{x+\sqrt{x}}}+\frac{1}{4\sqrt{x+\sqrt{x}}\sqrt{x}}\right]$$

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QUIZ 6

NAME: ____________________

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1. (10 MARKS) Find dy/dx by implicit differentiation:

$$xsiny + ysinx = 1$$

$(x)'\sin y + x(\sin y)' + (y)'\sin x + y(\sin x)' = 0$

$\sin y + xy'\cos y + y'\sin x + y\cos x = 0$

$xy'\cos y + y'\sin x = -\sin y - y\cos x$

$y'(x\cos y + \sin x) = -\sin y - y\cos x$

$$y' = \frac{-\sin y - y\cos x}{x\cos y + \sin x}$$

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Quiz 7

Name: ______________________

UtorID: ______________________

1. (10 Marks) Use the guidelines to sketch the graph:

$$y = (x-3)\sqrt{x} = x^{3/2} - 3x^{1/2}$$

Guidelines

A. Domain
B. Intercepts
C. Symmetry
D. Asymptotes
E. Intervals of inc/dec.
F. Local Max/Min
G. Concavity / POI
H. Sketch!

A. $x \geq 0,\ x \in \mathbb{R}$. 1

B. $\underline{x=0}$ $\quad \underline{y=0}$ $\quad (0,0)$

1 $\underline{y=0}$ $\quad 0 = (x-3)\sqrt{x}$

$\underline{x=3}$ $\quad (3,0)$

C. $f(-x) = (-x-3)\sqrt{-x}$ x, neither. 1

D. None. 1

E. $y' = \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2} = 0$

$y'(\frac{1}{4}) = \frac{3}{2}(\frac{1}{2}) - \frac{3}{2}\cdot 2$ 1

$= \frac{3}{4} - 3 = -$

$x^{1/2} = x^{-1/2}$

$\sqrt{x} = \frac{1}{\sqrt{x}}$

$x = 1$

	$x=0$	($\frac{1}{4}$)	$x=1$	(10)
y		↘		↗
y'		$-$ve		$+$ve.
		dec $(0,1)$		inc $(1,\infty)$

1

F. $x=1$ local min.

$y = -2$ 1

$(1,-2)$

G. $y'' = \frac{3}{4}x^{-1/2} + \frac{3}{4}x^{-3/2} = 0$

$x^{-1/2} = -x^{-3/2}$

1 $\quad \frac{1}{\sqrt{x}} = -\frac{1}{\sqrt{x^3}}$

$1 = -x^{1/2 - 3/2}$

$1 = -x^{-1}$

$1 = -\frac{1}{x}$

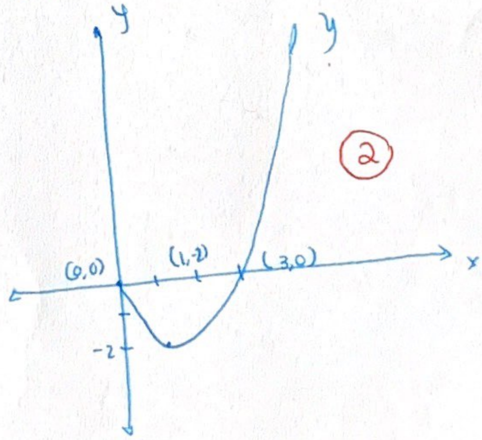
$x = -1 \rightarrow$ not in dom!

no POI

$y'' \geq 0$

$\therefore$ concave up $(0,\infty)$

H.



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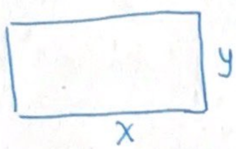
Quiz 8

Name: ____________________

UtorID: ____________________

1. (10 Marks) Find the dimensions of a rectangle with area $1000m^2$ whose perimeter is as small as possible.



$A = xy = 1000$

$y = \frac{1000}{x}$ (1)

$P = 2y + 2x$ ~~max~~ minimize. (2)

$P = \frac{2000}{x} + 2x = \frac{2000 + 2x^2}{x}$ (2)

$P'(x) = \frac{4x \cdot x - (2000 + 2x^2)}{x^2} = \cancel{\frac{2x^2 - 2000x}{x^2}} \cancel{\neq 2 - \frac{2000}{x}}$

(1)

$= \frac{2x^2 - 2000}{x^2} = 0$ (1) $\rightarrow$ $2x^2 = 2000$

$x = \pm\sqrt{1000}$, but length cannot be -ve

$\therefore \quad x = \sqrt{1000}$ (1)

$y = \frac{1000}{\sqrt{1000}}$ (1) $= 1000^{+1/2} = \sqrt{1000}$

Rectangle (square) of dimensions.

$\sqrt{1000}$ m by $\sqrt{1000}$ m (1)

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Quiz 9

Name: ______________________

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1. (10 Marks) Use the definition of the integral to evaluate (Hint: Use Riemann Sum):

$$\int_1^4 (x^2 - 4x + 2)dx$$

Check: $\left[\frac{x^3}{3} - 2x^2 + 2x\right]_1^4 = \left[\frac{64}{3} - 32 + 8 - \frac{1}{3} + \cancel{2} - \cancel{2}\right] = 21 - 32 + 8 = -3$

$\Delta x = \frac{4-1}{n} = \frac{3}{n}$ 2 $\quad x_i = 1 + \frac{3}{n} i$

2 $= \lim\limits_{n\to\infty} \sum_{i=1}^{n} \left[(x_i)^2 - 4x_i + 2\right]\frac{3}{n} = \lim\limits_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\left(1+\frac{3i}{n}\right)^2 - 4\left(1+\frac{3i}{n}\right) + 2\right]$

$= \lim\limits_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[1 + \frac{6i}{n} + \frac{9i^2}{n^2} - 4 - \frac{12i}{n} + 2\right] = \lim\limits_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\frac{9i^2}{n^2} - \frac{6i}{n} - 1\right]$

$= \lim\limits_{n\to\infty} \frac{3}{n} \left[\frac{9}{n^2} \sum_{i=1}^{n} i^2 - \frac{6}{n} \sum_{i=1}^{n} i - \sum_{i=1}^{n} 1\right]$ 2 $= \lim\limits_{n\to\infty} \frac{3}{n} \left[\frac{\cancel{9}^{3}}{n^2} \cdot \frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}_{2}} - \frac{\cancel{6}^{3}}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{\cancel{2}} - n\right]$ 2

$= \lim\limits_{n\to\infty} \frac{3}{n} \left[\frac{6n^2 + 3n + 6n + 3}{2n} - 3n - 3 - n\right] = \lim\limits_{n\to\infty} \frac{3}{n} \left[3n + \frac{9}{2} + \frac{3}{2n} - 4n - 3\right]$

$= \lim\limits_{n\to\infty} \frac{3}{n} \left[-n + \frac{3}{2} + \frac{3}{2n}\right] = \lim\limits_{n\to\infty} -3 + \frac{9}{2n} + \frac{9}{2n^2}$ 1

$= -3.$ 4